Derivatives

Introduction to Derivatives

Spot Market → A market where → Assets → Are bought and sold → For immediate delivery

Derivative Market → A market where → Contracts are made → To Buy and sell assets → At a future date for predetermined price

Meaning of Derivative

Derivative → Is a financial instrument → Whose value is derived → From value of an underlying asset → Underlying asset eg : Shares, Commodities, Currencies, Indices or Interest rates

Derivatives
- Futures
- Options

Example → A call option on Tata Motors its Strike Price ₹700 for 3months

TATA MOTORS

After 3 Months

If Share Price = ₹750

Holder Exercises the Option

Buys @ 700, gains = ₹50

If Share Price = ₹680

Option is not exercised

Value = ₹0

Thus the value of the derivative depends on the underlying asset

Expectations in Market

In any Market → There are only two possibilities

Price may increase

Price may decrease

Accordingly , investors take different Positions using derivatives

Positions Based on Expectations

If Price is Expected to increase

- Buy in Spot Market (eg: Tata Motors Shares)
- Take a long position in Forward Contract
- Take a long position in Futures Contract
- Buy a call option (eg: Tata Motors 700 CE)
- Sell a put Option

If Price is Expected to Decrease

- Short sell in Spot Market
- Take a Short Position in Forward Contract
- Take a Short Position in Futures Contract
- Sell a Call option
- Buy a put option

Types of Market Participants

1. **Hedgers**
   - Aim to Reduce or eliminate risk
   - Purpose = protection not profit
2. **Speculators**
   - Aim to earn profit by taking risk
   - In reality may gain or lose
3. **Arbitragers**
   - Identify price differences in different markets and exploit them
   - Correctly executed >> risk free profit

Hedging = **Expectation & Fear**

Expectation	Price will increase	Price will Decrease
Fear	What if it decreases?	What if it increases?

This Combination of expectation & fear leads to hedging

Where an investor uses derivatives to protect against unfavourable price moments

Futures Contract

It is an agreement → Entered today → To buy or sell →

- A Currency or
- A Share or
- A Commodity

→ At a future date for a pre agreed price

Positions in Futures

Long Position

Taken when the expectation is that the Price will increase

Objective : Profit if Price rises

Short Position

Taken when the expectation is that the Price will Decrease

Objective : Profit if Price falls

Closing a Futures Contract

To Close a futures Contract at maturity, an investor takes the Opposite Position

Long

- Short to close or
- sell

Short

- long to Close or
- Buy

✅ Long Position

Asset : Tata Motors Shares

Futures Contract :

Buy 100 Shares at ₹700 each, delivery in 1month (long Position)

Expectations : Price will increase

Possible outcomes at maturity

Profit

Market price = ₹750

Close the Contract by Selling at ₹750

Profit = (₹750 -₹700) x 100 = ₹5,000

Price increased ⬆️ as expected

Profit realized 💵

Loss

Market price = ₹680

Close the Contract by selling at ₹680

Loss = (₹700 – ₹680) x 100 = ₹2,000

Price decreased ⬇️ against expectation

Loss incurred 🤦‍♀️😭

🔑 key point → Profit if price rises , loss if price falls

✅ Short Position

Asset : Tata Motors Shares

Futures Contract :

Buy 100 Shares at ₹700 each, delivery in 1month (short Position)

Expectations : Price will Decrease

Possible outcomes at maturity

Profit

- Market price = ₹680
- Close the Contract by buying at ₹680
- Profit = (₹700 −₹680) x 100 = ₹2,000
- Price Decreased ⬇️ as expected
- Profit realized 💵

Loss

- Market price = ₹750
- Close the Contract by Buying at ₹750
- Loss = (₹750 − ₹700) x 100 = ₹5,000
- Price increased ⬆️ against expectation
- Loss incurred 🤦‍♀️😭

🔑 key point → Profit if price falls , loss if price raises

Futures

- Mark to market Account (MTM)
  - 1. Initial Margin
  - 2. Maintenance Margin
  - 3. Variation Margin
- Fair futures price & Arbitrage
- Hedging
  - Cross hedging → Taking an appropriate position in index
  - Selling high risk shares & Buying low risk shares
  - Buying risk free securities
    - Fresh investment
    - Sell existing shares proportionately & buy risk free securities
    - Sell the shares in order of risk & buy risk free securities
  - Commodity Hedging

Mark to Market account

A daily Settlement System → In futures trading → Where profit or loss → Is Calculated at the end of each Trading day

Ensures that → Gains / losses → Are credited/ debited daily → Reducing default risk

Logic

Futures prices → Fluctuates daily → Instead of waiting for Contract expiry → MTM → Ensures each traders account → Reflects Current market value

Example → Long 100 Tata motors futures at ₹700 → Price today = ₹710 → Gain = (710-700) x 100 → = ₹1000 → Credited to MTM account

Initial Margin

Upfront amount → Deposited by a trader → To enter a futures Contract

Acts as security for potential losses

Formula → Initial margin = $\mu + 3\sigma$

Logic for formula → Initial margin = Daily absolute Change + 3 x standard Deviation

- Daily absolute Change : Covers normal day to day price movement
- 3 x Standard Deviation : Covers extreme price movements

→ Ensures that most price fluctuations are covered and reduces default risk

Maintenance Margin

The minimal balance → That must be maintained → In MTM Account → To keep the futures position active

The minimal balance → If MTM balance falls below this → Margin call is triggered

Example → Initial margin = ₹10,000
Maintenance margin = ₹7000
→ MTM losses reduce balance to ₹6500
→ Trader must deposit Variation margin

Variation Margin

Additional funds → Deposited to bring → MTM Account → Back to initial Margin level after losses

Fair futures price

The price at which → Futures Contract → Should trade → So that no arbitrage opportunity exists

Formula

Fair futures price = Spot price + Carrying Cost – yield/income

Or

$$\text{Fair futures price} = \text{Spot price} \times e^{tr} - \text{yield}$$

Or

$$\text{Fair futures price} = \text{Spot price} \times e^{t(r-y)}$$

Logic

- Spot price : current market price of the asset
- Carrying Cost : Cost to hold the asset until expiry (interest, storage, insurance
- Yield / income : any benefits from the asset (dividends for shares, yield for bonds)
- Idea : Futures Price reflects Cost of Carry minus any benefits , so traders cannot earn risk free profits

Case	Futures market	Action
AFP=FFP	Correctly valued	No scope for arbitrage
AFP>FFP	Over valued	Sell future , Buy spot
AFP<FFP	Undervalued	Buy future, sell spot

Example → Gold futures

Spot price : ₹60,000 per 10 grams

Carrying cost (interest/Storage for 1month) : ₹500

Yield/Benefit : Nil

Fair futures price → 60000 + 500 - 0 = ₹60,500

	Arbitrage
If the futures Contract is above ₹60,500	Sell futures, buy spot
If the futures Contract is below ₹60,500	Buy futures, sell Spot

Hedging with index future

Formulas

$$\beta_P = \frac{\text{\% of change in portfolio}}{\text{\% of change in market}}$$

Beta +ve → Market & Securities moves Same direction

Beta -ve → Market & Securites moves in Opposite direction

To find number of Contracts

- Target Beta not given
- Target beta given

$$\frac{\text{Portfolio} \times \beta_p}{\text{Value of future Contracts}}$$

$$\frac{\text{Portfolio value (existing.}\beta - \text{target.}\beta)}{\text{Value of future Contracts}}$$

- β_p ⬆ – Sold
- β_p ⬇ – Buy

Perfect Hedging → Example → Infosys Shares with nifty futures

Step 1 : Scenario setup

You own : 1000 Infosys shares

Current Infosys Price : ₹3300 per share

↓

Portfolio Beta : 1.2 (Infosys moves 1.2 x market)

↓

Expectation : price will rise

Fear : price may decrease → Hedge with nifty futures

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Nifty futures price : ₹25,000

Lot size. : 10

Step 2 → Calculate Futures Contracts for Complete Hedge

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Futures position value = portfolio value x Beta

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Portfolio value = 3,300 x 1,000. = 33,00,000

Futures position value = 33,00,000 x 1.2

= 39,60,000

$$\text{Number of Contracts} = \frac{\text{portfolio value} \times \text{Beta}}{\text{value of future contracts}}$$

$$= \frac{39{,}60{,}000}{25000 \times 10} = 16 \text{ Contracts}$$

✅ perfect whole Number of Contracts ,,,, net gain /loss = 0

Step 3 : Market moves

Case 1	Case 2
Market falls by 10%	Market raises by 10%
Infosys portfolio loss : 3,96,000 Nifty futures gain : 3,96,000	Infosys portfolio gain : 3,96,000 Nifty futures loss : 3,96,000
Net effect 0	Net effect 0

Step 4 : key Takeaways

Hedging with index futures → Protects your stock portfolio from market risk

Perfect hedge formula

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Futures position value = portfolio value x Beta

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Outcome : any gain / loss in stock is exactly offset loss / gain in futures

Introduction to options

Meaning of options

An option is a → Financial derivative → That gives → Right ✅ → but not the obligation ❎

To buy or sell → An underlying asset → At a pre determined price → On or before a specific date

✍ Types of Options

- **Call option** → Right to buy the underlying asset at a strike price
- **Put option** → Right to sell the underlying asset at a strike price

Exercised	FSP > X	FSP < X
Break even point	FSP = X+P	FSP = X-P
Holder	Unlimited Profit Limited loss	Profit = X-P Limited loss
Writer	Limited profit unlimited loss	Profit = premium Loss = X-P

Example

Call option on HDFC Bank
Strike price ₹1700 , Expiry 3months

If after 3 months → HDFC Bank = ₹1850

Holder exercises call → Buys at ₹1700 → Profit = ₹150/share

If HDFC Bank < ₹1700 → Option not exercised → Value = 0

Put option on ICICI Bank
Strike price ₹900, Expiry 3 months

If after 3 months → ICICI Bank = ₹800

Holder exercises put → Sells at ₹900 → Profit = ₹100/ shares

If ICICI Bank > ₹900 → Option not exercised → Value = 0

Put call parity

Put call parity → It is a fundamental relationship → Between the prices of → European Call option & European Put Option → On the same underlying asset → With the same strike price & expiry date

Purpose → It ensures | No arbitrage opportunity | Exists in the market

I.e, | You cannot make a risk free profit by exploiting mispricing between call and put options

Equation

$$C + PV(K) = P + S$$

Where

- C = call option price / minimum Value of call
- P = put option price / minimum value of put
- K = spot price of the underlying asset
- S = strike price
- P = present value of strike price

Minimum Value of Call (C)

Spot price − $X \cdot e^{-trf}$

Minimum Value of put (P)

$X \cdot e^{-trf}$ − spot price

Interpretation in Simple words

Cost of buying a call option and investing PV of strike price

=

Cost of buying a put option and buying the underlying asset

Key idea : If this relationship doesn't hold, she does scan on a risk free profit by arbitrage

Option Valuation

Option valuation → Determines → The fair price today

So that the holder knows the → Maximum premium to pay

And the writer knows → Minimum premium to charge

Binomial Model

Binomial model

- Write a call option
- Calculate hedge ratio
- Hold Δ shares to reduce loss
- Calculate PV of portfolio
- Calculate value of call / put

$$\frac{\Delta \text{ in value of option}}{\Delta \text{ in future spot price}}$$

1. Construct a risk free portfolio

You write a call option and hold a certain number of shares (Δ) of underlying

Δ is chosen such that the combined portfolio's value does not depend on the future price of underlying

2. Future value is Certain

Because the portfolio is hedged with Δ shares

No matter how the underlying move, the portfolio has a known value at maturity

This removes the uncertainty

The portfolio is effectively risk free

3. Discount to present

Since a portfolio is risk free

It's future value can be discounted at the risk free rate to get its present value

4. Derive the option value

The portfolio today consist the call option and Δ shares

If you subtract the value of Δ shares today,
the remaining portion is the fair value of the call option

Risk - Neutralization method

To know the exact value of call & put

Probability of FSP

$$(P \times UFSP) + (1-P) \times DFSP = \text{spot price} \times e^{trf}$$

Assume investors are risk neutral

Expect the risk free return

Future spot price can go up or down

Assign probabilities (up) ⬆️ and (down) ⬇️

Calculate call values for up and down scenarios

Discount expected call value at risk free rate

It gives fair price of option today

Two period Binomial Model

1. Step 1 - Construct Price tree

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The underlying asset can move up ⬆️ or down ⬇️ in each period

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Build a two step binomial tree of possible future prices

2. Step 2 - Calculate Risk - Neutral Probability

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Use the risk free rate to determine

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Neutral probability of an upward move ⬆️ (P)

3. Step 3 - compute option value at maturity

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At the final nodes (end of two periods)

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Calculate the option pay off for each scenario

4. Step 4 - work backwards to present value

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Use risk neutral probabilities
to calculate the expected value at the previous node

↓

Repeat until you reach today's date

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Gives the present value of the option

Black scholes Model

In the black scholes → We assume continuous price movement

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The stock can move anywhere, not just two discrete possibilities

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Volatility is used to model the expected range of price movement

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Works only for European options (exercise at maturity)

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Assumes no dividends are paid during the options life

Formula

Step 1 : $d_1 = Ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)(t) / \sigma \times \sqrt{t}$

Step 2 : $d_2 = d_1 - (\sigma \times \sqrt{t})$

Step 3 : $n(d_1)$

Step 4 : $n(d_2)$

Step 5 : V_c = Spot price (nd_1) – Strike price $\times e^{-trf}(nd_2)$

Step 6 : V_p = Strike price $\times e^{-trf} n(-d_2)$ – Spot price $\times n(-d_1)$

$$n(-d_1) = 1 - nd_1$$
$$n(-d_2) = 1 - nd_2$$

- Sp = Spot price
- X = Strike Price
- Ln = natural Log
- r = rate of interest
- σ = standard deviation
- t = time

Greek Words

- **Delta**: Change in option price w.r.t change in Stock price
- **Gamma**: Change in delta W.r.t Change in stock price
- **Vega**: Change in option price w.r.t volatility
- **Rho**: Change in option price w.r.t interest rates
- **Theta**: Change in option price w.r.t time delay

		Strike price	Expiry date
Strangle	Buying a call & put	Different	Same
Straddle	Buying & selling a call & put option	Same	Same
Horizontal option spread	Buying & selling call or put option	Same	Different